Small strain shear modulus for compacted sand-clay mixtures

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**Abstract**

The dynamic properties of soils are required to analyze their behavior under earthquake loading. Shear modulus is the important input parameter in dynamic analysis. A series of resonant column tests was carried out on pure clays and sand-clay mixtures prepared at different densities to investigate the effects of sand content, clay plasticity and void ratio on the shear modulus in small and medium shear strain. On the basis of the resonance column test results, a mathematical model was developed for the maximum shear modulus. The reduction in shear modulus with increasing shear strain for mixture with different sand content studied. In addition the effect of clay plasticity on shear modulus reduction curves in small and medium shear strain investigated. 

\[ G/G_{\text{max}} = 1/(1+\gamma/\gamma_d)^\alpha \]

for sand-clay mixture used in this study.

**Keywords:** Shear Modulus, Damping Ratio, Sand-clay Mixture, Small Strain

1. Introduction

Adequate information on dynamic soil properties, especially dynamic shear modulus, is essential for accurate computations of ground response and soil-structure interaction problems. Compacted aggregate-clay mixtures have been successfully used as the cores of embankment dams. These materials, called composite clays by Jafari and Shafiee (2004), are usually broadly graded and are composed of clay as the main body with sand, gravel, cobble or even boulders floating in the clay matrix. The most important factors that influence include shear strain amplitude, mean effective confining stress, soil type and plasticity index, frequency of loading, and number of loading cycles. Earlier studies on shear modulus for composite gravel-clay soils did not show this complex effect of plasticity index on damping ratio (Seed and Idriss 1971; Vucetic and Dobry 1991).

Although composite soils with properties between cohesive and granular materials are found in nature enormously, unlike sands and clays, less effort has been dedicated toward understanding their dynamic behavior. This is mainly due to the inherent difficulties in characterizing heterogeneous media. The investigation described in this paper entails a study on the low-amplitude dynamic properties of sand-clay mixtures using resonant column tests. Low, medium and high plastic clays were mixed with different amounts of sand to explore the effect of the soil plasticity and aggregate content on the dynamic properties. Eventually, on the basis of the 72 test results, a mathematical model for the maximum shear modulus is presented. Strain dependency of shear modulus and damping is simply modeled. Effect of sand content, initial shear modulus and soil plasticity are also considered in the modeling.
2. TESTED MATERIALS AND PROCEDURE

2.1. Materials Tested

Low, medium and high plastic pure clays with nine mixtures of the clays with sand were used in this study. Fig.1 and Table 1 present the grain-size distribution, and physical properties for the clays and sand. The sand used in the study was retrieved from a riverbed and composed of surrounded particles with minimum and maximum void ratios of 0.655 and 0.901 respectively, and a specific gravity of 2.66. Fig. 1 shows the grain-size distribution for the sand. As seen in Fig.1, all the sand particles are greater than 0.5 mm, and they will all remain on the 425µm sieve. Thus, the plasticity index of sand-clay mixtures will be that of the clay portion when it is determined on the basis of the ASTM D4318-05 (2005).

![Grain-size distribution for the clays and sand](image)

Fig 1. Grain-size distribution for the clays and sand

<table>
<thead>
<tr>
<th>Clay Type</th>
<th>Plasticity Index, PI (%)</th>
<th>G_s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Plastic (CL)</td>
<td>12</td>
<td>2.72</td>
</tr>
<tr>
<td>Medium Plastic (CM)</td>
<td>25</td>
<td>2.70</td>
</tr>
<tr>
<td>High Plastic (CH)</td>
<td>50</td>
<td>2.71</td>
</tr>
</tbody>
</table>

2.2. Specimen Preparation

The specimen preparation technique was chosen to model as precisely as possible the in situ condition of the core materials of embankment dams. All the specimens, typically 70 mm in diameter and 100 mm in height were prepared, with three different void ratio and water content of 2% wet of optimum. Appropriate amounts of clay and sand for each layer were first thoroughly mixed. Each layer was then mixed with water at least 24 hours before use and sealed. The material was poured in six layers into a cylindrical mold and compacted. To achieve a greater uniformity of specimens, a procedure similar to the undercompaction technique (Ladd, 1978) was used. For each layer, the compactive effort was increased toward the top by increasing the number of blows per layer. Each layer was then scored after it was compacted for better bonding with the next layer.

2.3. Test procedure

The specimens were saturated with a Skempton B value in excess of 97%. To facilitate the saturation process, CO2 was first percolated through the specimens then de-aired water was flushed into the specimens. Lastly, a back pressure of 150 kPa was incrementally applied to accelerate the saturation rate. The specimens were then isotropically consolidated under effective confining stresses of 300 and 500 kPa. Following consolidation, torsional resonant column tests were carried out under the specifications of ASTM D 4015 (2003), using a fixed-free type device.
3. A MATHEMATICAL MODEL FOR $G_{\text{max}}$

The maximum shear modulus, $G_{\text{max}}$ was determined as the shear modulus in very small strain range where shear modulus was almost constant and the amount of excess pore pressure was negligible small. Many experimental investigations carried out on sandy and normally consolidated clayey soils in early studies [Drnevich and Richart, 1970; Seed et al., 1986] showed $G_{\text{max}}$ was basically related to the mean effective principal stress, $\sigma'_{\text{m}}$ and void ratio, $e$ expressed by the well known equation:

$$G_{\text{max}} = A F(e).\left(\frac{\sigma'_{\text{m}}}{P_a}\right)^n.p_a$$

(1)

in which $A$ is an empirical constant reflecting soil fabric formed through various stress and strain histories, $P_a$ is atmospheric pressure, $n$ is empirically determined exponent, approximately 0.4-0.6 (Bobby and Kalinski, 2005; Zhou and Chen, 2005), and $F(e)$ is void ratio function, which is usually given by:

$$F(e) = \frac{(B - e)^2}{1 + e}$$

(2)

where constant B is usually taken as 2.97 (Hardin and Richart, 1963; Zhou and Chen, 2004).

In order to characterize the low-amplitude dynamic properties of aggregate-clay mixtures, it was necessary to find an appropriate mathematical model in the form of Eq. (1). Herein, a regression analysis based on the least square technique was used to find the values of constants $A$, $n$ and $B$ in Eqs.(1) and (2). The analyses were carried out by varying $B$ and $n$ for each mixture until achieving a coefficient of determination, $R^2$ more than 95%. Analyses of data showed, regardless of the density and plastic properties of the mixture, $G_{\text{max}}$ can be successfully related to $\sigma'_{\text{m}}$ and $e$ by taking $n=0.5$ and $B=2.95$. Values of $A$ for each mixture calculate. Figure 3 shows the variation of $A$ against sand content. As seen, constant $A$ linearly increases with aggregate content until a sand content of 60%. As the sand content continues to increase above 60%, $A$ decreases. Thus, $A$ can be described as a function of sand content (SC) by the following linear equations (Figure 2):

$$A = 1700SC + 2000 \quad SC \leq 60\%$$

(3)

$$A = -2000SC + 4300 \quad 60\% < SC$$

(4)

So the following equation can be used to determine $G_{\text{max}}$. It is interesting to note that the equation of $G_{\text{max}}$ in sand-clay mixtures is independent of the clay plasticity. It can be say, $e$ represent the effect of plasticity index and porosity of samples.
4. MODELING OF $G/G_\text{max}$–$\gamma$

It is customary to represent the variation in shear modulus at shear strain level by normalizing it with maximum shear modulus. This facilitates a comparison of the relationship of soils of different type. Properties of the pure clay and clay-sand mixture samples are shown in table 2. In this table clay plasticity (PI), sand content (SC) and the name of soil type are shown.

Table 2. Properties of the pure clay and clay-sand mixture samples

<table>
<thead>
<tr>
<th>Soil type</th>
<th>PI</th>
<th>Agg</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0</td>
<td>12.5</td>
<td>0</td>
</tr>
<tr>
<td>M0</td>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>H0</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>L40</td>
<td>12.5</td>
<td>40</td>
</tr>
<tr>
<td>M40</td>
<td>25</td>
<td>40</td>
</tr>
<tr>
<td>H40</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>L60</td>
<td>12.5</td>
<td>60</td>
</tr>
<tr>
<td>M60</td>
<td>25</td>
<td>60</td>
</tr>
<tr>
<td>H60</td>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td>L80</td>
<td>12.5</td>
<td>80</td>
</tr>
<tr>
<td>M80</td>
<td>25</td>
<td>80</td>
</tr>
<tr>
<td>H80</td>
<td>50</td>
<td>80</td>
</tr>
</tbody>
</table>

Hyperbolic models have been widely used to describe nonlinear soil behavior under cyclic loading (e.g., Hardin and Drnevich 1972; Pyke 1993; Stokoe et al. 1999). Improved fits to the test data can be obtained using modified hyperbolic models, similar to the model suggested by Stokoe et al. (1999). The following equation is proposed in this study to approximate the strain dependency of $G/G_\text{max}$.

$$G/G_\text{max} = \frac{1}{1 + \left(\gamma/d\right)^{\alpha}}$$

(6)

$\gamma_d$ and $\alpha$ are two constant parameters for every soil type (12 type of soil tested). The $G/G_\text{max}$ values of sample of clay-sand mixture and pure clay at two confining pressure are plotted together against $\gamma$ in figs. 3(a) and 3(b). Curves of Eq. (6) fitted for the data are also shown in these figs 3(a). The values of $\gamma_d$ and $\alpha$ for every soil type are shown in Table 4.2. Results of tests shows that samples with higher clay plasticity have maximum shear modulus for a wider range of shear strains when compared with samples of low clay plasticity. $G/G_\text{max}$ decrease with increase of $S.C.$ until $S.C. = 40\%$ and $G/G_\text{max}$ for $S.C. = 40\%~60\%$ show similar value. Beyond $S.C. = 60\%$, $G/G_\text{max}$ decrease with increase of $S.C.$ up to $80\%$. Moreover, samples with low fines content have lower normalized shear modulus with the increase in strain level.

Figure 4 show the variation of $\gamma_d$ decrease with increase of $S.C.$ and increase with increase of $P.I$. Effect of plasticity index and sand content on $\gamma_d$ in pure clay is more then the effect of them in mix samples. All sand-clay mixture show similar effect of PI (variation of $\gamma_d$ in term of PI show similar slope); this figure show $\gamma_d$ decrease with increase of sand content.

Figure 5 show the variation of $\alpha$ in term of PI for pure clay samples and sand-clay samples. The variation of $\alpha$ in term of PI is same for all samples, $\alpha$ increase with PI.

It is reported by several researchers that $\gamma_d$ in equation 6 is reference shear strain ($\gamma_t$ shear strain for $G/G_\text{max}=0.99$) but result of this researchers didn’t support this idea.
Zen et al. (1987) and Vucetic and Dobry (1991) have suggested $G/G_{max}-\gamma$ relations based on clay plasticity for clays. The expressions they proposed are very useful to easily obtain the dynamic properties of normally consolidated clays. This is because the clay plasticity is a basic parameter that best shows the physical description of the cohesive soil, and
it is possible to easily obtain this parameter from laboratory tests on disturbed or remolded samples. Wang and Kuwano (1999) showed that $G/G_{\text{max}}$–$\gamma$ relations based on sand content for clay-sand mixture. This study show that $\gamma_d$ is as a function of sand content and clay plasticity.

Table 3. Values of $\gamma_d$ and $\alpha$

<table>
<thead>
<tr>
<th>Soil</th>
<th>$\alpha$</th>
<th>$\gamma_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0</td>
<td>0.94</td>
<td>3.5E-04</td>
</tr>
<tr>
<td>M0</td>
<td>0.96</td>
<td>4.1E-04</td>
</tr>
<tr>
<td>H0</td>
<td>1.00</td>
<td>5.5E-04</td>
</tr>
<tr>
<td>L40</td>
<td>0.94</td>
<td>1.9E-04</td>
</tr>
<tr>
<td>M40</td>
<td>0.96</td>
<td>2.1E-04</td>
</tr>
<tr>
<td>H40</td>
<td>1.00</td>
<td>2.5E-04</td>
</tr>
<tr>
<td>L60</td>
<td>0.94</td>
<td>1.8E-04</td>
</tr>
<tr>
<td>M60</td>
<td>0.96</td>
<td>2.0E-04</td>
</tr>
<tr>
<td>H60</td>
<td>1.00</td>
<td>2.3E-04</td>
</tr>
<tr>
<td>L80</td>
<td>0.94</td>
<td>1.3E-04</td>
</tr>
<tr>
<td>M80</td>
<td>0.96</td>
<td>1.3E-04</td>
</tr>
<tr>
<td>H80</td>
<td>1.00</td>
<td>1.5E-04</td>
</tr>
</tbody>
</table>

It is reported by several researchers that confining pressure has either limited or no effect on the $G/G_{\text{max}}$–$\gamma$ relations of clay (Kokusho et al. 1982; Vucetic and Dobry 1991; Zen et al. 1978, 1987); however, confining pressure influences the behavior of sands (Kokusho 1980; Tatsuoka et al. 1978). Further study is needed on this.
6. CONCLUSIONS

An experimental study was performed on the compacted pure clays and mixtures of sand-clay to investigate the effect of sand content, confining stress, void ratio and plasticity index on the low-amplitude and medium-amplitude dynamic deformation properties using resonant column tests. A mathematical model was also developed for maximum shear modulus, $G_{\text{max}}$ of sand-clay mixtures. Variation of normalised shear modulus and damping ratio in term of shear modulus studied and a model for $G/G_0$, developed. The following conclusions may be drawn based on this experimental study:

1. $G_{\text{max}}$ increases with aggregate content, until a maximum $G_{\text{max}}$ is reached at a sand content of 60%. As the sand content continues to increase above 60%, $G_{\text{max}}$ decreases.

2. A mathematical model was developed for $G_{\text{max}}$ of the mixtures examined in this study. The model is very similar to Hardin and Drnevich (1972) model for normally consolidated clays and predicts $G_{\text{max}}$ as

$$A \left(\frac{2.95 - e}{1 + e}\right)^2 \sigma_m^{0.5}$$

where $A$ is a function of sand content, $e$ is void ratio and $\sigma_m$ is mean effective principal stress.

3. $G/G_{\text{max}}$ decreases with sand content but for $40\% \leq \text{S.C.} \leq 60\%$ decrease is negligible.

4. $G/G_{\text{max}}$ increases with confine pressure.

5. $G/G_{\text{max}}$ increase with increase of plasticity index for pure clay and sand-clay mixture but with increase of sand content effect of plasticity index decrease.

6. Samples with higher clay plasticity have maximum shear modulus for a wider range of shear strains when compared with samples of low clay plasticity.

7. Effect of plasticity index and sand content on $\gamma_d$ in pure clay is more then the effect of them in mix samples

8. All sand-clay mixture show similar effect of plasticity index.

9. Model of $G/G_{\text{max}} = 1/[1 + \gamma_d^{\alpha}]$ is useful to predicts $G/G_{\text{max}}$ for sand-clay mixture and pure clay.

10. $\gamma_d$ for model calculated in term of sand content and plasticity index. $\gamma_d$ in equation 6 is not reference shear strain ($\gamma_r$)

11. $\alpha$ increase with plasticity index, the variation of $\alpha$ in term of PI is same for all samples.

7. REFERENCES


